APS/123-QED

The total cross-sections for the photoeffect for K-shell bound electrons and pair production with the created electron in the ground state for photon energies above 1 MeV

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Considering the contributions of the main term of the relativistic Coulombian Green function given by Hostler to the second order of S matrix element and taking into account only the large components of the Dirac spinor of the ground state we obtain the imaginary part of the Rayleigh amplitudes in terms of elementary functions. Thereby simple and high accurate formulae for the total cross-sections for photoeffect and pair production with the electron created in the K-shell are obtained *via* the optical theorem.Comparing the predictions given by our formulae with the full relativistic numerical calculations of Kissel *et al* [Phys. Rev. A 22, 1970 (1980)] and Scofield [LLRL, Internal Report,1973], a good agreement is found for photon energies above the pair production threshold up to 5 MeV for any Z elements. We present our numerical results for the total photoeffect and pair production cross-sections, for various photon energies for the K-shell of Ag and Pb.

PACS numbers: 32.80.Cy

arXiv:physics/0606182 v3 2 Oct 2006

I. INTRODUCTION

The S matrix element for Rayleigh scattering of an initial photon with momentum $\overrightarrow{k_1} = \omega\overrightarrow{\nu}_1$and polarization vector $\overrightarrow{s}_1$, and a final photon with momentum $\overrightarrow{k_2} = \omega\overrightarrow{\nu}_2$and polarization vector $\overrightarrow{s}_2$, by a K-shell bound electron, if we take into account only the large components of the Dirac spinor of the ground state and only the main term of the relativistic coulombian Green function[1], is:[4]

$$\mathcal{M}_R = M(\omega,\theta)(\overrightarrow{s}_1\overrightarrow{s}_2) + N(\omega,\theta)(\overrightarrow{s}_1\overrightarrow{\nu}_2)(\overrightarrow{s}_2\overrightarrow{\nu}_1) \tag{1.1}$$

where θ is the photon scattering angle, and the invariant amplitudes M and N are:

$$M(\omega,\theta) = \mathcal{O} - P(\Omega_1,\theta) - P(\Omega_2,\theta) \tag{1.2}$$

$$N(\omega,\theta) = Q(\Omega_1,\theta) + Q(\Omega_2,\theta) \tag{1.3}$$

with the Rayleigh scattering amplitudes given by:

$$P(\Omega,\theta) = \frac{2^7\lambda^5X^3}{d^4(\Omega)}\frac{\omega\pm\omega_{pp}}{2m}\frac{F_1(2-\tau;2,2;3-\tau;x_1,x_2)}{2-\tau} \tag{1.4}$$

$$Q(\Omega,\theta) = \frac{2^{11}\lambda^5X^3\omega^2}{d^5(\Omega)}\frac{\omega\pm\omega_{pp}}{2m}\frac{F_1(3-\tau;3,3;4-\tau;x_1,x_2)}{3-\tau} \tag{1.5}$$

where

$$\Omega_1 = \gamma m + \omega, \Omega_2 = \gamma m - \omega = -|\Omega_2|,$$
$$\gamma = \left(1-\alpha^2Z^2\right)^{\frac{1}{2}}, \lambda = \alpha Z m, \omega_{pp} = (1+\gamma)m,$$
$$\tau_j = \frac{\alpha Z\Omega_j}{X_j}, X_j = m^2 - \Omega_j^2 \text{ with } \mathrm{Re}[X_j] > 0, j = 1,2$$

The function $\mathcal{O}$ is the atomic form factor, while $F_1(a;b_1,b_2;c;x_1,x_2)$ is the Appell hypergeometric function of four parameters and two complex variables given by the relationships:

$$x_1x_2 = p = \left[\frac{d^*(\Omega_1)}{d(\Omega_1)}\right]^2 = \xi^2 \tag{1.6}$$

$$x_1 + x_2 = s = 2\xi - \frac{16X^2\omega^2\sin^2\frac{\theta}{2}}{d^2(\Omega_1)} = \xi^2 \tag{1.7}$$

$$\text{with} \quad \begin{aligned} d(\Omega_j) &= 2m(\pm\gamma\omega - \alpha^2Z^2m + \alpha ZX_j), \\ d^*(\Omega_j) &= 2m(\pm\gamma\omega - \alpha^2Z^2m - \alpha ZX_j) \end{aligned} \tag{1.8}$$

In the equations (1.4)-(1.5) and (1.8) the upper sign corresponds to the case $\Omega = \Omega_1$, while the lower sign corresponds to the case $\Omega = \Omega_2$.

Obviously, in the case of the forward scattering, the amplitude $N(\omega,\theta)$ is no longer present in the Rayleigh amplitude. Also, the imaginary part of the amplitude $\mathcal{M}_R$ is given only by the terms $P(\Omega_1,0)$ and $P(\Omega_2,0)$ which are present in the expressions of the photoeffect and the pair production cross-sections respectively.

II. THE TOTAL CROSS-SECTION OF THE PHOTOELECTRIC EFFECT AND PAIR PRODUCTION IN THE CASE OF K SHELL

According to the optical theorem, the imaginary part of the Rayleigh amplitude for forward scattering allows to get the total photoeffect cross-section *per* K-shell electron:

$$\sigma_{ph} = \frac{4\pi}{\alpha}\frac{m}{\omega}r_0^2\left|\mathrm{Im}[P(\Omega_1,0)]\right| = \frac{16\pi^2}{3}r_0^2m^2\alpha^5Z^6\frac{\omega+\omega_{pp}}{2m}\frac{\Omega_1\left|X_1^2\right|}{\omega^5}\left(1+\left|\tau_1\right|^2\right)\frac{(-\xi_1)^{\tau_1}}{e^{\pi|\tau_1|}-e^{-\pi|\tau_1|}} \quad (2.1)$$

In a similar way we get the total pair production cross-section *per* K-shell electron:

$$\sigma_{pp} = \frac{4\pi}{\alpha}\frac{m}{\omega}r_0^2\left|\mathrm{Im}[P(\Omega_2,0)]\right| = \frac{16\pi^2}{3}r_0^2m^2\alpha^5Z^6\frac{\omega-\omega_{pp}}{2m}\frac{|\Omega_2|\left|X_2^2\right|}{\omega^5}\left(1+\left|\tau_2\right|^2\right)\frac{(-\xi_2)^{\tau_2}}{e^{\pi|\tau_2|}-e^{-\pi|\tau_2|}} \quad (2.2)$$

We point out that: $(-\xi_1)^{\tau_1} = e^{-|\tau_1|\chi_1}$, with

$$\chi_1 = \pi - 2\arctan\left(\frac{\alpha Z\left|X_1\right|}{\gamma\omega-\alpha^2Z^2m}\right); \omega > \frac{\alpha^2Z^2m}{\gamma} \quad (2.3)$$

and $(-\xi_2)^{\tau_2} = e^{-|\tau_2|\chi_2}$, with

$$\chi_2 = \pi - 2\arctan\left(\frac{\alpha Z\left|X_2\right|}{\gamma\omega+\alpha^2Z^2m}\right); \omega > \omega_{pp} \quad (2.4)$$

III. NUMERICAL RESULTS AND CONCLUSIONS

Using our analytical formulae for the cross sections for K-shell electrons we get the numerical numerical results in Table I, Table II, and figure 1 for the whole K-shell.

TABLE I: Photoeffect and pair production cross sections for Ag K-shell.

Energy (keV)	Pair production cross section (mb)	Photoeffect cross section (mb)	Cross sections ratio
1000	9.145x10^{-7}	493.836	5.399x10^{8}
1100	0.0875335	401.376	4585.39
1249	0.851678	306.868	360.309
1500	3.11888	211.705	67.878
2000	7.16782	122.626	17.107
2754	9.96235	70.2819	7.054
3000	10.3218	61.0716	5.916
3500	10.6112	47.8107	4.505
4000	10.5516	38.99	3.695
4807	10.1208	29.7828	2.942
5000	9.98901	28.1578	2.818
5500	9.62702	24.6344	2.558
6000	9.25496	21.8602	2.362
6500	8.88765	19.6248	2.208
7000	8.53296	17.7887	2.084
7500	8.1948	16.2559	1.983

As it may be observed from equations (2.3) and (2.4), for higher photon energies the arguments of the exponentials, χ_1 and χ_2, have the same limit, $\pi - 2\arctan[\frac{\alpha Z}{\gamma}]$, so that the two cross sections will become closer and closer as the photon energy increases.

Comparing the predictions given by our formulae with the full relativistic numerical calculations of Kissel *et al* [2] and Scofield[3], a good agreement is found for photon energies above the pair production threshold up to 5 MeV for any Z elements.

The good agreement of our calculation with the full relativistic results shows that, for the presented energies regime, the main relativistic kinematics terms are canceled by some retardation and multipols terms, and the spin effects are small.

ACKNOWLEDGEMENT

This work was partially supported by the Romanian National Research Authority (ANCS) under Grant CEEX PC-D11-PT00-582/2005.

TABLE II: Photoeffect and pair production cross sections for Pb K-shell.

Energy (keV)	Pair production cross section (mb)	Photoeffect cross section (mb)	Cross sections ratio
1000	0.11370	5099.07	44846.5
1100	2.36329	4151.89	1756.82
1249	12.2336	3181.11	260.03
1500	37.1725	2200.44	59.195
2000	79.4739	1278.86	16.091
2754	107.831	735.099	6.817
3000	111.338	639.19	5.741
3500	113.924	500.947	4.397
4000	112.949	408.876	3.619
4807	108.012	312.65	2.894
5000	106.549	295.652	2.774
5500	102.571	258.781	2.522
6000	98.517	229.731	2.331
6500	94.5392	206.311	2.182
7000	90.7119	187.065	2.062
7500	87.0731	170.993	1.963

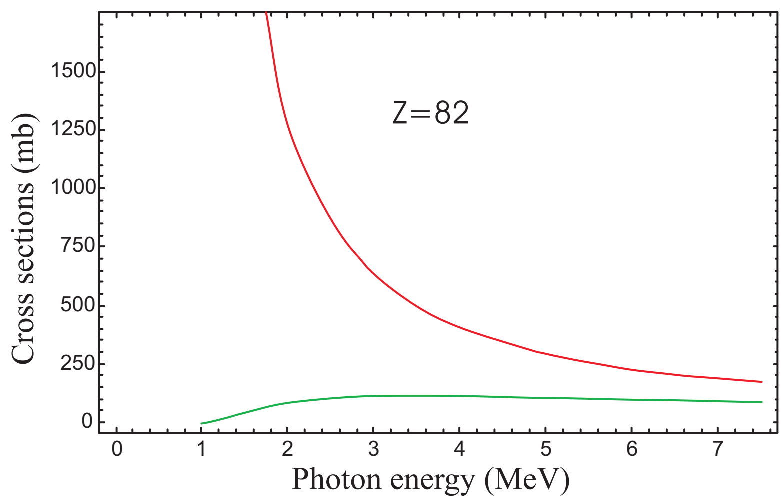

FIG. 1: Photoeffect (red) and pair production (green) cross sections for Pb K-shell

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